5.4.3 Centre of stiffness and elastic displacements of the diaphragm

The current paragraph examines the special case of orthogonal columns in parallel arrangement. The general case is examined in Appendix C.

5.4.3.1 Subject description

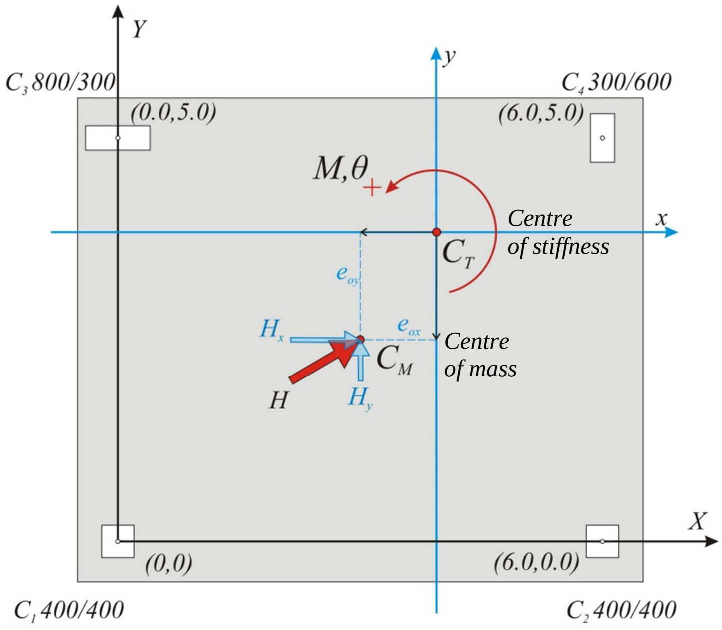

Figure 5.4.3.1-1: Simple one-storey structure comprising four columns, whose tops are connected by a rigid slab-diaphragm.

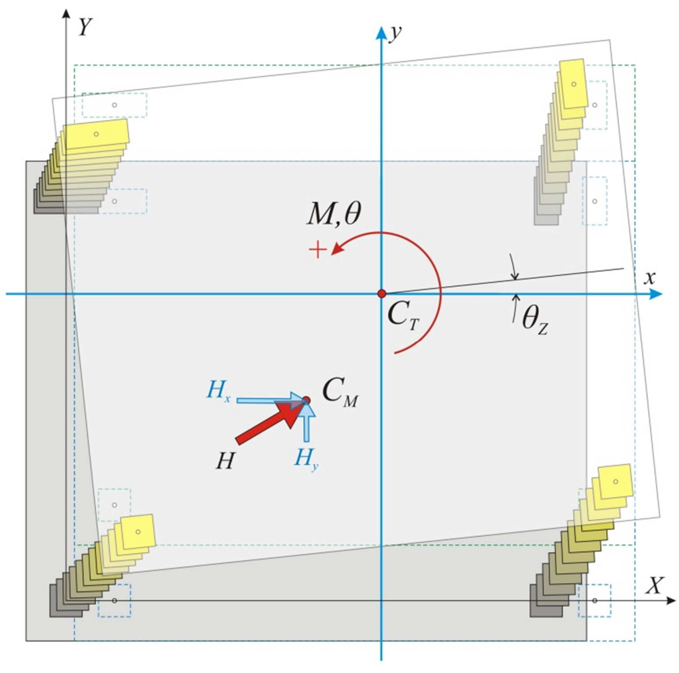

*Figure 5.4.3.1-2: Parallel translation of the diaphragm in both directions and rotation , due to a force **H** applied to the centre of mass **C_M** (**X0Y** initial coordinate system, **xC_Ty** main coordinate system)*

When a horizontal force ***H*** acts on a storey level, all the points of the slab including the column[9] tops move in accordance with the same rules due to the in-plane rigidity of the slab.

These rules induce the diaphragm to develop a parallel (translational) displacement by δ_{xo}, δ_{yo} and a rotation θ_z about the **centre of stiffness** $C_T(x_{CT}, y_{CT})$ in xC_Ty coordinate system, which is parallel [10] to the initial coordinate system X0Y and has as origin the point C_T.

The diaphragmatic behaviour may be considered as a superposition of three cases:

(a) parallel translation of the diaphragm along the X direction due to horizontal force component H_X,

(b) parallel translation of the diaphragm along the Y direction due to horizontal force component H_Y,

(c) rotation of the diaphragm due to moment M_{CT} applied at the centre of stiffness C_T.

The horizontal seismic forces are applied at each mass point, while the resultant force is applied at the centre of mass C_M.

In case the direction of the force H passes through the point C_T as well as C_M the moment has zero value and therefore the diaphragm develops zero rotation.

5.4.3.2 Translation of centre of stiffness C_T along x direction

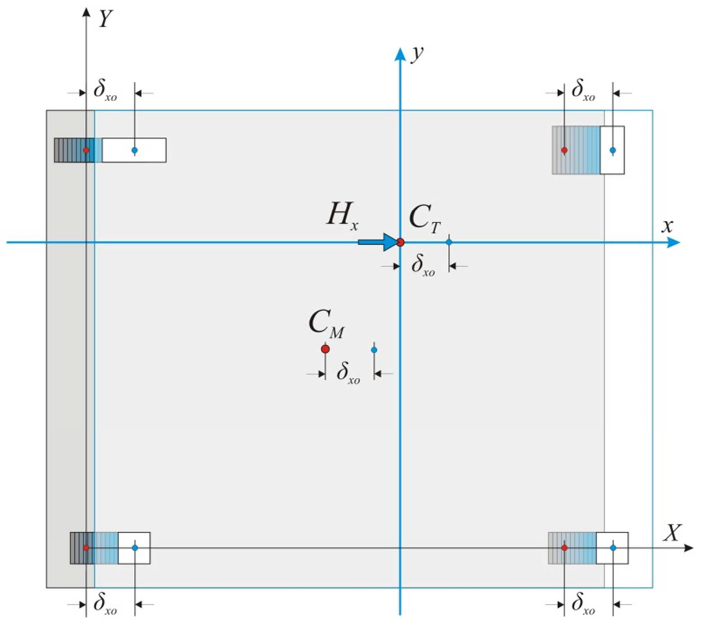

Figure 5.4.3.2: Parallel translation along the x direction due to force H_x applied at C_T

[9] Henceforth the term ‘column’ accounts for terms column and wall.

[10] In the general case, i.e. in the case of columns with inclined local principal axes with respect to the initial system X0Y, the inclination angle of the principal system with respect to the initial system is a≠0˚ (see Appendix C). Therefore, when the system of orthogonal columns is parallelly arranged then $K_X=K_x$, $V_X=V_x$, $K_{XY}=K_{xy}=0$, meaning that a horizontal force applied at the centre of stiffness in x direction results in a displacement only along x (the same applies for y direction).

In case a horizontal force H_x is applied at C_T in x direction, the following 2 equilibrium equations apply:

- The sum of forces in x direction is equal to H_x, i.e. $H_x=\Sigma(V_{xoi})$ (i).
- The sum of moments V_{xoi} about the point C_T is equal to zero, i.e. $\Sigma(V_{xoi}\cdot y_i)=0$ (ii).

Each column i carries a shear force $V_{xoi}=\delta_{xo}\cdot K_{xi}$.

$\Sigma(V_{xoi})=\Sigma(\delta_{xo}\cdot K_{xi})=\delta_{xo}\cdot\Sigma(K_{xi})$, expression (i) gives $H_x=\delta_{xo}\cdot\Sigma(K_{xi}) \rightarrow$

$H_x=K_x\cdot\delta_{xo}$ *where* $K_x=\Sigma(K_{xi})$.

Expression (ii) gives $\Sigma(V_{xoi}\cdot[Y_i\text{-}Y_{CT}])=0 \rightarrow \Sigma(V_{xoi}\cdot Y_i)\text{-}\Sigma(V_{xoi}\cdot Y_{CT})=0 \rightarrow Y_{CT}\cdot\Sigma(V_{xoi})=\Sigma(V_{xoi}\cdot Y_{CT}) \rightarrow$

$Y_{CT}=\Sigma(\delta_{xo}\cdot K_{xi}\cdot Y_i)/\Sigma(\delta_{xo}\cdot K_{xi}) \rightarrow Y_{CT}=\Sigma(K_{xi}\cdot Y_i)/\Sigma(K_{xi})$

5.4.3.3 Translation of centre of stiffness C_T along y direction

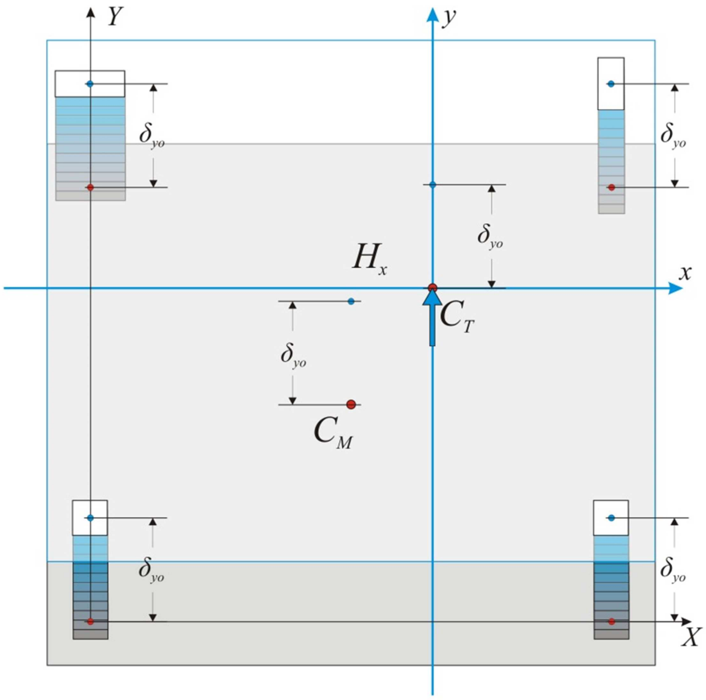

Figure 5.4.3.3: Parallel translation in y direction due to force H_y applied at C_T

Accordingly, the corresponding expressions are derived for direction y.

$H_y=K_y\cdot\delta_{yo}$ where $K_y=\Sigma(K_{yi})$ and $X_{CT}=\Sigma(K_{yi}\cdot X_i)/\Sigma(K_{yi})$

Summarising, the centre of stiffness and the lateral stiffnesses are defined by the following expressions:

<u>Centre of stiffness and lateral stiffnesses:</u>

$$X_{CT}=\frac{\sum(X_i\cdot K_{yi})}{\sum(K_{yi})},\ H_x=K_x\cdot\delta_{xo}\ \text{ where }\ K_x=\sum(K_{xi}) \qquad (4')$$

$$Y_{CT}=\frac{\sum(Y_i\cdot K_{xi})}{\sum(K_{xi})},\ H_y=K_y\cdot\delta_{yo}\ \text{ where }\ K_y=\sum(K_{yi}) \qquad (5')$$

5.4.3.4 Rotation of the diaphragm by an angle θ_z about C_T

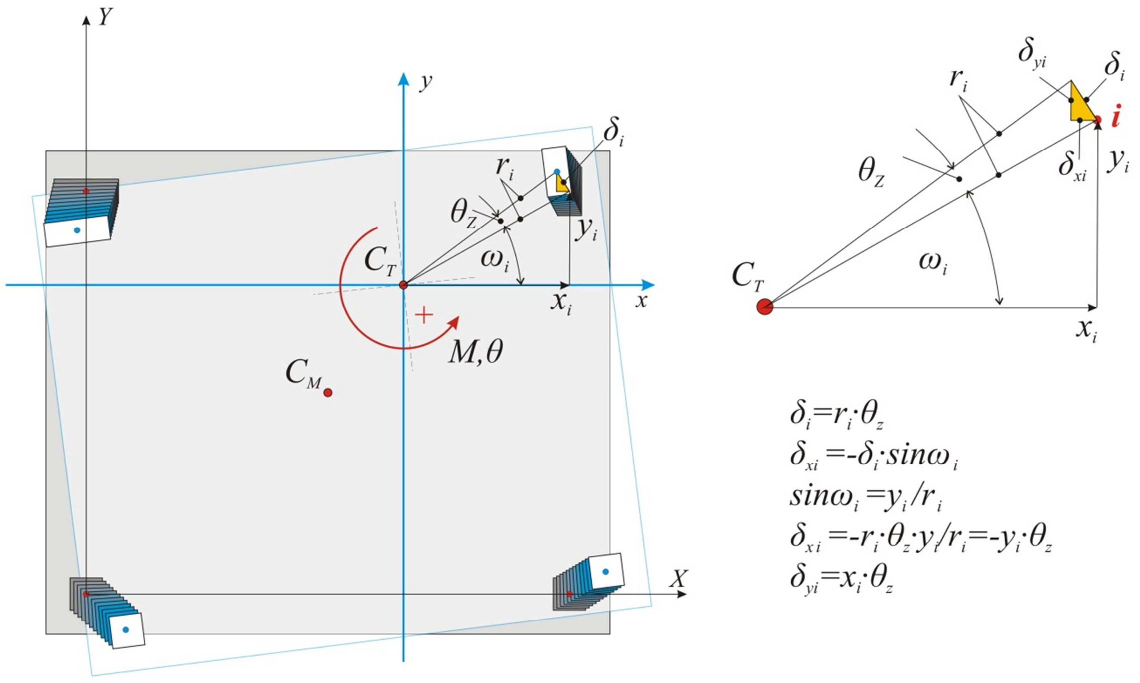

Figure 5.4.3.4: Displacements due to rotation developed from moment M applied at C_T

To determine the deformation developed by external moment M, applied at the centre of stiffness C_T, the initial system X0Y is transferred (by parallel translation) to the principal system xC_Ty. The centre of mass is transferred to the principal system along the structural eccentricities[11] e_{ox}, e_{oy} in accordance with the following expressions:

Principal coordinate system

$$x_i = X_i - X_{CT}\,,\; y_i = Y_i - Y_{CT}\,,\; e_{ox} = x_{CM}\,,\; e_{oy} = y_{CM} \qquad (6')$$

The displacement of the diaphragm consists essentially of a rotation θ_z about the C_T, inducing a displacement δ_i at each column top i with coordinates x_i,y_i in respect to the coordinate system with origin the C_T. If the distance between the point i and the C_T is r_i, the two components of the (infinitesimal) deformation δ_i are equal to $\delta_{xi}=-\theta_z\cdot y_i$ and $\delta_{yi}=\theta_z\cdot x_i$.

The shear forces V_{xi} and V_{yi} in each column developed from the displacements δ_{xi}, δ_{yi} are:

$V_{xi}=K_{xi}\cdot\delta_{xi}=K_{xi}\cdot(-\theta_z\cdot y_i) \rightarrow V_{xi}=-\theta_z\cdot K_{xi}\cdot y_i$ and $V_{yi}=K_{yi}\cdot\delta_{yi}=K_{yi}\cdot(\theta_z\cdot x_i) \rightarrow V_{yi}=\theta_z\cdot k_{yi}\cdot x_i$

The resultant moment of all shear forces V_{xi}, V_{yi} about the centre of stiffness is equal to the external moment M_{CT}, i.e.

$M_{CT}=\Sigma(-V_{xi}\cdot y_i+V_{yi}\cdot x_i+K_{zi}) \rightarrow M_{CT}=\theta_z\cdot\Sigma(K_{xi}\cdot y_i^2+K_{yi}\cdot x_i^2+K_{zi})$

Torsional stiffness K_{zi} of column i

Columns resist the rotation of the diaphragm by their flexural stiffness expressed in terms $K_{xi}\cdot y_i^2$, $K_{yi}\cdot x_i^2$ (in $N\cdot m$), and their torsional stiffness K_{zi}, which is measured in units of moment e.g. $N\cdot m$.

[11] The eccentricities e_{ox}, e_{oy} are called structural because they depend only on the geometry of the structure and not on the external loading. As presented in chapter 6, besides structural eccentricities, accidental eccentricities also exist.

5.4.4 Assessment of building torsional behaviour

The degree of the torsional stiffness of a diaphragmatic floor is determined by the relation between the equivalent mass inertial ring (C_M, l_s) and the torsional stiffness ellipse (C_T, r_x, r_y). The optimal location of the two curves is where the torsional stiffness ellipse encloses the mass inertial ring.

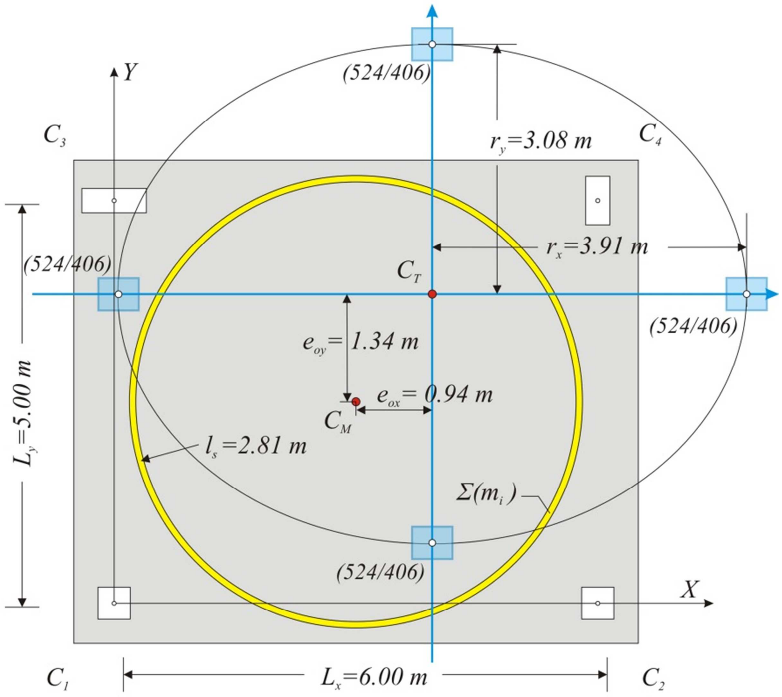

Figure 5.4.4: Equivalent mass inertial ring (C_M, l_s) and torsional stiffness ellipse (C_T, r_x, r_y)

A building is classified as torsionally flexible *[EC8 §5.2.2.1]* if either $r_x<l_s$ or $r_y<l_s$ is satisfied in at least one diaphragm storey level. In this example both conditions are satisfied.

For a building to be categorized as being regular in plan, the two structural eccentricities e_{ox}, e_{oy} at each level shall satisfy both conditions $e_{ox}\leq 0.30r_x$ & $e_{oy}\leq 0.30r_y$ *[EC8 §4.2.3.2]*. In this particular example the first condition is satisfied $e_{ox}=0.94\ m \leq 0.30r_x\ (=0.30\times 3.91=1.173\ m)$, whereas the second one is not $e_{ox}=1.34\ m \leq 0.30r_x\ (=0.30\times 3.08=0.924\ m)$. Therefore the building that comprises that specific floor diaphragm is not regular in plan.

Simplified seismic analysis may be performed, provided that the following conditions are met for each x, y direction:

$r_x^2 > l_s^2 + e_{ox}^2$

$r_y^2 > l_s^2 + e_{oy}^2$ *[EC8 §4.3.3.1(8) d)]*.

In this example the first condition is satisfied

$3.91^2(=15.3)>2.81^2+0.94^2(=7.9+0.9=8.8)$,

whereas the second one is not

$3.08^2(=9.5)<2.81^2+1.34^2(=7.9+1.8=9.7)$.

We therefore conclude that the simplified seismic analysis may not be performed at the building including this particular floor diaphragm.

Calculation[22] **of the diaphragmatic behaviour**
1st (and unique) **floor level**

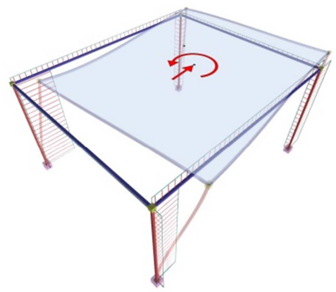

Figure 5.4.5.4-2

1st Loading:

H_X=90.6 kN

eccentricity[23] c_Y=1.0 m

$M_{CM,X}$=90.6 kNm

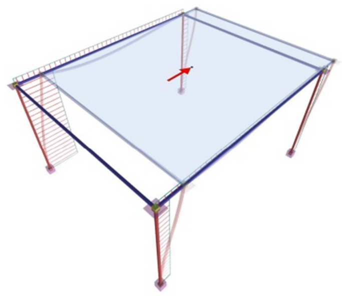

Figure 5.4.5.4-3

2nd Loading:

H_X=90.6 kN

Diaphragm restrained against rotation

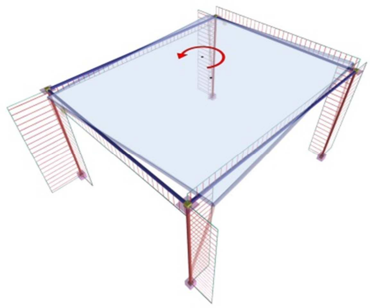

Figure 5.4.5.4-4

*(**1st Loading**) minus (**2nd Loading**):*

H_X=0

$M_{CT,X}=90.6 \cdot y_{CM}+90.6 \cdot c_Y$

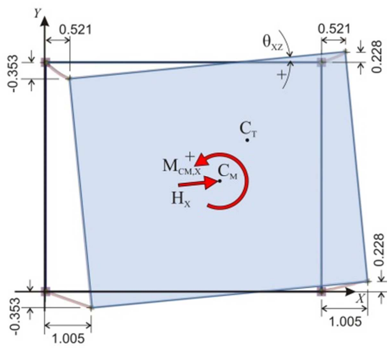

Figure 5.4.5.4-5

The displacements of each point i

$\delta_{X,i}$, $\delta_{Y,i}$

and the rotation angle of the diaphragm

$\theta_{XZ}=9.681\times10^{-5}$

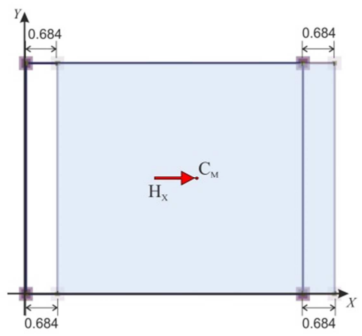

Figure 5.4.5.4-6

The diaphragm develops zero rotation and moves parallelly[24] *to the axes X, Y.*
Each point of the diaphragm (therefore the C_T as well) has the same principal displacements
δ_{XXo}=0.684 mm, δ_{XYo}=0.

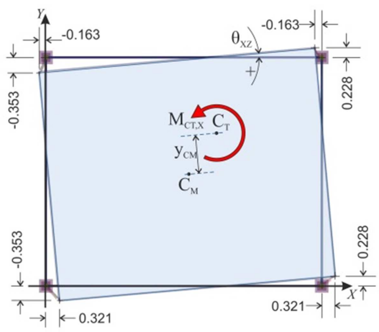

Figure 5.4.5.4-7

The diaphragm develops only a rotation θ_{XZ} about C_T.
The displacements of each point i due to rotation are equal to:
$\delta_{Xt,i}=\delta_{X,i}-\delta_{XXo}$, $\delta_{Yt,i}=\delta_{Y,i}-\delta_{XYo}$.
The C_T derives from the expressions:
X_{CT}[25] $=X_1-\delta_{Yt,1}/\theta_{XZ}=3.646$ m
$Y_{CT}=Y_1+\delta_{Xt,1}/\theta_{XZ}=3.316$ m

---

22 The analysis of the diaphragmatic floor is performed automatically by the software. Algorithms are verified using the tools provided by the software. In this example with zero angle *a* of the principal system, all the diaphragm data may be calculated by two simple analyses and by the equations of the special case a=0, already presented in the previous paragraphs. Here, the general case of columns arranged randomly is been used, which applies even in the special case of the rectangular columns in parallel arrangement. The method is explained in detail in Appendix D.

23 The horizontal seismic load is applied at the C_M. The eccentricity of the loading can be given also as equivalent torsional moment $M_{CM,X}=H_X \cdot c_Y$, which in this case is equal to $M_{CM,X}=90.6\times1.0=90.6$ kNm. This additional eccentricity aims to increase the effect of the rotation, i.e. to give larger displacements due to rotation, in order to calculate the torsion related data of the diaphragm more accurately.

24 In the special case of an one-storey building comprising only rectangular columns arranged parallelly to the axes X,Y, the horizontal force acting in X or Y displaces the diaphragm only in X or Y.

25 The equations determining the C_T coordinates are general and may be applied for each point.
Indicatively, for column 4:

Calculation of the diaphragmatic behaviour (continued)
1st (and only) **floor level**

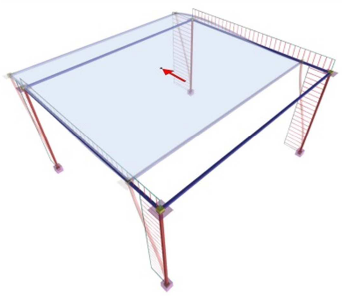

Figure 5.4.5.4-8

3rd Loading:

H_Y=90.6 kN
Diaphragm restrained against rotaion

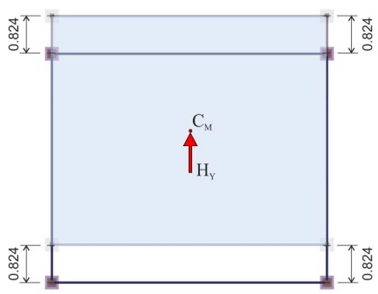

Figure 5.4.5.4-9

Analysis results:

The diaphragm is not rotated, but only translated in parallel to the axes X, Y.

Each point of the diaphragm (therefore and the C_T) has the same principal displacement:

$\delta_{YXo}=0,\ \delta_{YYo}=0.824$ *mm.*

The 3 rd analysis completes the necessary series of analyses for the determination of all diaphragm data.

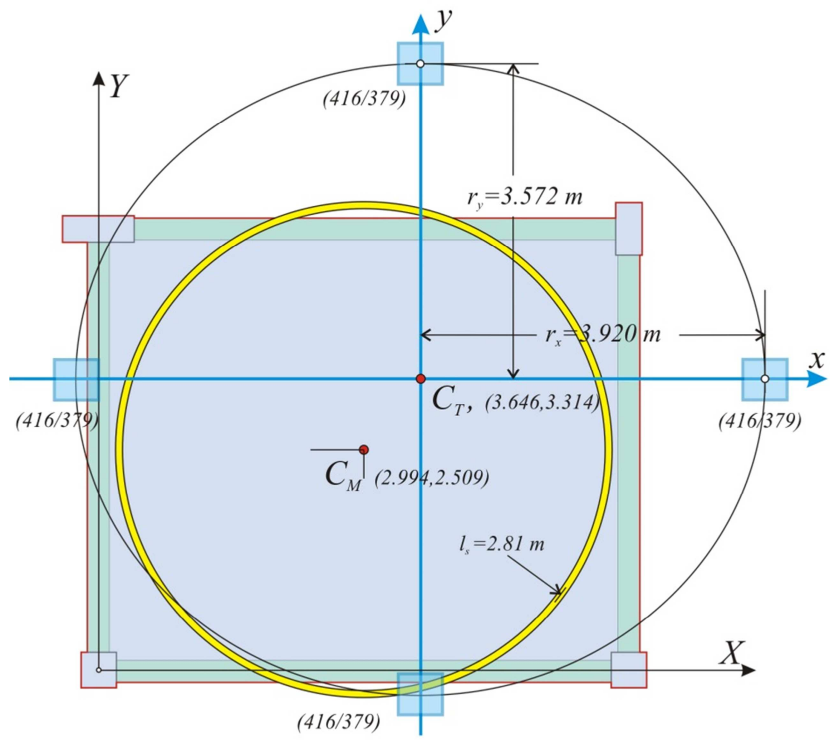

Figure 5.4.5.4-10

Definition of the principal system[26], of the torsional stiffness radii and of the equivalent system *(see §C.6):*

$$\tan(2a)=2\delta_{XYo}/(\delta_{XXo}-\delta_{YYo})=0.0 \rightarrow 2a=0° \rightarrow a=0°$$

$$\delta_{xxo}=\delta_{XXo}=0.684\ mm,$$

$$\delta_{yyo}=\delta_{YYo}=0.824\ mm$$

$$K_{xx}=H_x/\delta_{xxo}=90.6\times10^3 m/0.684\times10^{-3}m=132.5\times10^6\ N/m$$

$$K_{yy}=H_y/\delta_{yyo}=90.6\times10^3 m/0.824\times10^{-3}m=110.0\times10^6\ N/m$$

$$M_{CT,X}=90.6\cdot y_{CM}+90.6\cdot c_Y=90.6\times(3.316-2.500)+90.6\times1.0=164.5\ kNm$$

$$K_\theta=M_{CT,X}/\theta_{XZ}=164.5/9.681\times10^{-5}=17.0\times10^5 kNm$$

$$r_x=\sqrt{K_\theta/K_{yy}}=\sqrt{17.0\times10^8 N/m/110.0\times10^6 N/m}=3.931m$$

$$r_y=\sqrt{K_\theta/K_{xx}}=\sqrt{17.0\times10^8 N/m/132.5\times10^6 N/m}=3.582\ m$$

$X_{CT}=X_4-\delta_{Yt,4}/\theta_{XZ}=6.0-0.228\times10^{-3}m/(9.681\times10^{-5})=6.0-2,355=3.645\ m$
$Y_{CT}=Y_4+\delta_{Xt,4}/\theta_{XZ}=5.0-0.163\times10^{-3}m/(9.681\times10^{-5})=5.0-1.684=3.316\ m.$

26 In this example, it is already determined that the angle of the principal system is zero, if the type of the structure is considered and the 2nd analysis (according to which δ_{XYo}=0). The calculation has been performed for the sake of generality. To this end, other quantities have also been calculated, such as the centre of stiffness, which in this case is obtained from the simple application of moment at the point C_M.